## Midterm 2 Solutions

1. (12 points) What is the IRR of the following cash flows?
a) You pay $\$ 990$ in year 0 and receive $\$ 1000$ in year 1 .
b) You pay $\$ 990$ in year 1 and receive $\$ 1000$ in year 2 .

A: In both cases $\operatorname{IRR}=1000 / 990-1 \approx 1 \%$
2. (18 points) Consider the following cashflow. You receive $\$ 100 \mathrm{k}$ today and then you make 10 annual payments of $\$ 12 \mathrm{k}$ each with the first a year from today.
a) What is the NPV when the MARR is $10 \%$ ?
$A: N P V=100 k-12 k / 10 \%\left(1-1.1^{\wedge}-10\right)=\$ 26,265$.
b) As a function of the discount rate, does the NPV increase, decrease, or both?

A: Increase, because it's the opposite of an investment.
c) Is the IRR $>10 \%$ ?

A: No, because of NPV $>0$ at $10 \%$ and NPV increasing.
3. (18 points) You bought a $\$ 200 \mathrm{k}$ condo. You got a 15 -year fixed-rate mortgage and made a $20 \%$ down payment.
a) What is your monthly payment?

A: principal $=80 \% 200 \mathrm{k}=160 \mathrm{k}$. Payment $=160 \mathrm{kr} /\left(1-(1+\mathrm{r})^{\wedge}-180\right)$. Here r is the monthly interest rate, the annual rate divided by 12 .
b) Would the monthly payment be bigger or smaller with a 30 -year mortgage at the same interest rate?

A: smaller.
c) To evaluate the NPV of the mortgage, is a real or actual discount rate more convenient? Explain.

A: Actual discount rate; because the nominal cashflows are constant.
4. (6 points) Will the nominal payments on a mortgage increase over time with inflation?

A: No, they are constant.
5. (6 points) Will the nominal rent payments increase over time with inflation (consider a long horizon, say 10 years)?

A: Yes.
6. (7 points) When would the actual discount rate be less than the real discount rate, $\mathrm{d}_{\mathrm{A}}<$ $\mathrm{d}_{\mathrm{R}}$ ?

A: When $\mathrm{f}<0$, since then $1+\mathrm{f}<1$ and thus $1+\mathrm{d}_{\mathrm{A}}=\left(1+\mathrm{d}_{\mathrm{R}}\right)(1+\mathrm{f})<1+\mathrm{d}_{\mathrm{R}}$.
7. (12 points) In which of the following is the assumption of repetition appropriate?
a) Deciding whether to reopen a copper mine.
b) Deciding whether to speed up R\&D for a new drug.
c) Deciding whether to replace the transmission on your car or to upgrade to a new car?
d) Deciding whether to buy Dell or Apple laptops for your company.

A: d. For c, you won't repeatedly replace the transmission (other things will also fail).
8. (21 points) You go for an oil change. Your mechanic offers to use premium oil (then the oil change would cost $\$ 35$ instead of $\$ 30$ for regular oil), which would mean you'd only need to come back in 9 mo . instead of 6 mo .
a) Suppose your MARR is $5 \%$. It's convenient for this problem to have a monthly discount rate. What is the equivalent monthly discount rate?

A: We search for $r$, so that $(1+r)^{12}=1.05$. Thus $r=1.05^{1 / 12}-1=0.41 \%$.
b) Suppose the monthly discount rate is $0.5 \%$. Determine whether it is more advantageous to use premium oil or regular oil. Assume an infinite horizon and repetition.

A: $\mathrm{r}=0.5 \%$. EUAC for regular oil $=30 \mathrm{r} /\left(1-(1+\mathrm{r})^{-6}\right)=\$ 5.09$.
EUAC for premium oil $=35 \mathrm{r} /\left(1-(1+\mathrm{r})^{-9}\right)=\$ 3.99$.
Using premium oil is more advantageous.
c) Would it be more convenient if the discount rate in part $b$ were a real or nominal discount rate? Explain.

A: Real, because the cash-flows are inflation-adjusted (the nominal costs will increase over time).

